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LETTER TO THE EDITOR

Alfvén wave reflection at a density transition region

J A Adam

Department of Applied Mathematics, University of St Andrews, North Haugh, St Andrews KY16 9SS, Fife, Scotland

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Abstract. It is pointed out that reflection and transmission coefficients may easily be obtained for Alfvén waves normally incident upon the solar transition region, by a suitable analytic choice of a density profile which is a good approximation to reality.

The purpose of this letter is to point out an interesting mathematical result concerning the propagation of Alfvén waves in a stratified compressible medium. The problem arises in the context of solar physics, the astrophysical consequences of which have been discussed elsewhere (Adam 1974), and its nature will be briefly described here. The density structure of the solar atmosphere is known to be very complex (Newkirk 1967), but in general terms it decreases, as one would expect, monotonically with altitude from the solar chromosphere to the corona. There does exist, however, a so called transition region between these two regions across which the density decreases very rapidly in a distance small compared to a typical coronal length-scale. Since the gas pressure is nearly constant across such a region there is a corresponding increase in temperature. For further details the paper by Newkirk (1967) should be consulted.

The fundamental wave equation governing motion in an atmosphere permeated by a magnetic field $\boldsymbol{B}_0(z)$ was originally derived by Ferraro and Plumpton (1958):

$$\frac{\partial^2 \boldsymbol{v}}{\partial t^2}=\nabla(c_0^2\nabla \cdot \boldsymbol{v}-gv_z)+\frac{[\nabla\times\nabla\times(\boldsymbol{v}\times\boldsymbol{B}_0)]\times\boldsymbol{B}_0}{4\pi\rho_0(z)}+\left(\frac{1}{g}\frac{\mathrm{d}c_0^2}{\mathrm{d}z}+(\gamma-1)\right)\boldsymbol{g}\nabla \cdot \boldsymbol{v}. \tag{1}$$

Here $\boldsymbol{v}=(v_x, v_y, v_z)$ is the Eulerian velocity perturbation, $\boldsymbol{g}=(0, 0, -g)$ is the acceleration due to gravity, $\rho_0(z)$ is the density distribution (assumed continuous) and $c_0(z)$ is the velocity of sound in the medium. The ratio of specific heats is denoted by γ.

For a uniform vertical magnetic field $\boldsymbol{B}_0=(0, 0, B_0)$ it may be easily shown (Ferraro and Plumpton 1958) that the equation for Alfvén wave propagation is

$$\frac{\mathrm{d}^2U}{\mathrm{d}z^2}+\frac{\omega^2U}{v_\mathrm{a}^2}=0 \tag{2}$$

where $v_y=U(z)\,\mathrm{e}^{\mathrm{i}(k_0x-\omega t)}$, $v_\mathrm{a}^2=B_0^2/4\pi\rho_0$, the quantity v_a being the Alfvén velocity. Equation (2) may be solved for an isothermal atmosphere in terms of Bessel functions, but for the present problem we proceed differently. It was shown by Brekhovskikh (1960) that by a suitable choice of variables the hypergeometric equation

$$\frac{\mathrm{d}^2F}{\mathrm{d}\xi^2}-\frac{(a+b+1)\xi-c}{\xi(1-\xi)}\frac{\mathrm{d}F}{\mathrm{d}\xi}-\frac{ab}{\xi(1-\xi)}F=0 \tag{3}$$

can be reduced to the form (2) above. It is well known that it is possible to analytically continue the solution of (3) as $z \to -\infty$ (in terms of the variable ξ) from the solution as $z \to \infty$, and hence to obtain reflection and transmission coefficients for particular classes of profiles $g(z)$ $(=\omega^2/v_a^2(z)$ here).

We point out here that it is possible to choose an analytic function for $g(z)$ which is a very good approximation after suitable normalization to the behaviour of the quantity $\rho_0(z)$ in the solar atmosphere (see Jordan 1965). This choice enables a comparision to be made between the reflection coefficients for a model of this kind and one of the simple contact discontinuity type, as a function of wavelength.

By defining the density profile as

$$\rho_0(z) = \rho_{z=-\infty}\left(1 - \frac{N\,e^{mz}}{1+e^{mz}}\right) \qquad (4)$$

we obtain the required form for the quantity $g(z) \propto 1 - [N\,e^{mz}/(1+e^{mz})]$. The behaviour of $v_a^2 \sim \rho_0^{-1} \sim g^{-1}$ is illustrated schematically in figure 1, where the point $z = 0$ occurs in the middle of the transition region. This is reasonable since the dimensions of the region are small compared to coronal and even upper chromospheric scale-heights.

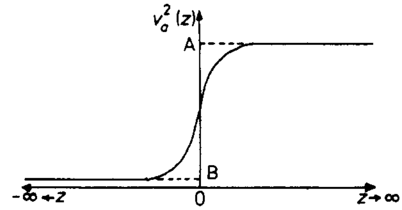

Figure 1. A schematic representation of the behaviour of $v_a^2(\propto \rho_0^{-1})$ through the transition region for constant B_0. A, $(v_a^2)_{\rm phot}/(1-N)$; B, $(v_a^2)_{\rm phot}$. The term $(v_a^2)_{\rm phot}$ refers to the value of v_a^2 at the photosphere ($z = -\infty$ in this model).

The constant N is related to the relative density drop across the layer, while m^{-1} is essentially a relative measure of the layer thickness. Using a result of Brekhovskikh in the context of the present problem, the energy reflection coefficient for normally-incident Alfvén waves at transition region is

$$P = \frac{\sinh^2[\frac{1}{2}\pi S(1-\sqrt{1-N})]}{\sinh^2[\frac{1}{2}\pi S(1+\sqrt{1-N})]} \qquad N<1 \qquad (5)$$

and $S = 2k_0/m$, k_0 being the initial wavenumber of the wave. In the limit of the contact discontinuity model, i.e. when $S \to 0$,

$$P \to \left(\frac{1-\sqrt{1-N}}{1+\sqrt{1-N}}\right)^2 \qquad (6)$$

—a standard result.

It has been shown (Adam 1974) that for all wavelengths of practical interest in solar physics, the contact-discontinuity model is a good approximation in these régimes to this more sophisticated model, a result which is not initially obvious.